VOL. 25, NO. 12 JOURNAL OF CLIMATE AND APPLIED METEOROLOGY DECEMBER 1986

# Prediction of Vessel Icing*

J. E. OVERLAND, C. H. PEASE AND R. W. PREISENDORFER
*Pacific Marine Environmental Laboratory/NOAA, Seattle, WA 98115*

A. L. COMISKEY**
*Arctic Environmental Information and Data Center, Anchorage, AK 99501*

(Manuscript received 7 September 1985, in final form 8 March 1986)

## ABSTRACT

Vessel icing from wave-generated spray is a severe hazard to expanded marine operations in high latitudes. Hardships in making observations during operations, combined with differences in vessel type and heading, have resulted in great variability in vessel icing observations for similar meteorological conditions. This has led to difficulties in development of quantitative forecast procedures. A categorical algorithm for relating vessel icing potential to wind speed, and air and sea temperatures is presented which seeks to minimize these difficulties. A set of 85 icing observations were collected for Alaskan waters from intermediate size vessels (20–75 m) during 1979 to 1983, and verified by interviews with the vessel operators and by comparison with National Weather Service analyses. Of the set of 85, 58 cases were open-ocean observations where the vessel was not heading downwind; 25% of this reduced set had icing rates in excess of 2.0 cm $h^{-1}$. Icing rate nomenclature and predicted icing rates for a given set of meteorological parameters developed from this study, and recommended for operations, are similar to those developed by Soviet and Japanese authors, but are five times greater than those based on the classic study by Mertins. This disparity is probably related more to differences in data analysis than to real geographic differences in icing conditions.

## 1. Introduction

Inability to forecast vessel icing is one of the most important marine meteorological problems in high-latitude waters because rapid accretion on decks and superstructures (Fig. 1) creates an extreme hazard for vessels from lack of stability (Shellard, 1974; Jessup, 1985). Severe icing conditions require the presence of subfreezing air temperatures, strong winds, and sea surface temperatures which are not more than 6°C above freezing (Vasilyeva, 1971). The region of icing often moves at such a rate that vessels cannot take evasive action unless they have prior warning. With increased emphasis on marine forecasts at high latitudes due to increased economic expansion and with general improvements in intermediate range atmospheric forecast models, there is potential for improvement in surface wind and air temperature forecasts. If these fields are combined with a suitable icing algorithm, icing rates can be forecast on a regional basis. Icing forecasts would be particularly valuable for planning marine operations since they would predict the movement of regions of potential icing hazard (Feit, 1985; MacDonald and Jessup, 1985).

Actual icing potential is a characteristic of each vessel, which depends on its design and sea-keeping ability. Differences in vessel type, combined with difficulties in making observations during operations, result in great variability in vessel icing observations for similar meteorological conditions (Stallabrass, 1980; Brown and Agnew, 1985a). This paper seeks to minimize these difficulties by developing a categorical algorithm for relating vessel icing to meteorological parameters.

An initial set of 195 icing incidents from Alaskan waters from 1979 to 1983 was reduced to a data set of 85 verified observations in which the individual vessel operators were contacted and interviewed. Meteorological information in these reports was also compared for consistency with the Anchorage Weather Service Forecast Office meteorological analyses. We define potential icing rate as the maximum sustained rate for typical Alaskan vessels, 20–75 m in length, which are not actively avoiding icing through heading downwind, moving at slow speeds, or avoiding open seas. Fifteen observations had icing rates greater than 2.0 cm $h^{-1}$; these rates are substantially greater than those in the limited Alaskan observation base available to Wise and Comiskey (1980) and the data base for eastern Canada (Stallabrass, 1980), similar to those quoted by Kachurin

---

* Contribution No. 773 from NOAA/Pacific Marine Environmental Laboratory.

** Present affiliation: Northern Technical Services, Anchorage, AK 99501.

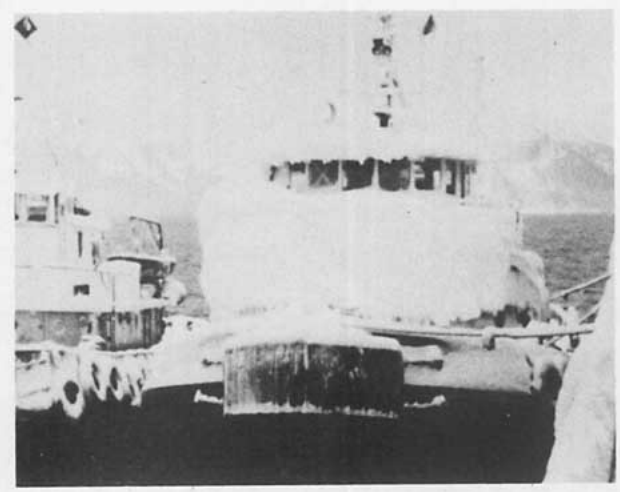


FIG. 1. A moderate case of vessel icing on the ocean-going tug *Crusader* (photograph by Capt. E. Guchee).

et al. (1974) for the Sea of Japan, Barents Sea and Bering Sea, Lundqvist and Udin (1977) for the Baltic Sea, and Sawada (1973) for the Sea of Okhotsk. The quantitative definitions for different qualitative icing terminology adopted by this study are similar to the definitions used by Kachurin et al. (1974) and other authors, but are more extreme than those used by Mertins (1968) (Table 1).

A major feature of the algorithm development is the use of a robust statistical procedure to relate icing rates to meteorological parameters. An index for prediction of potential icing rate from meteorological variables is developed based on current understanding of the physical process of icing. A priori categorical limits are established for light, moderate, and heavy (severe) icing rates as in Table 1. Bivariate pairs are formed for each observation between the icing category and the value of the predictor. The value of the predictor for the transition point between icing categories is then determined based on an objective measure which maximizes the number of correct forecasts (Preisendorfer, 1983). Finally, an independent measure of the skill of the algorithm is obtained. The method is considered robust because the influences of inaccuracies in any individual observation in the data set is minimized by basing the algorithm upon icing and predictor categories. This contrasts with regression techniques in which extreme observations either have undue weight or are excluded from the data set as outlying values. The technique should have application to prediction of secondary meteorological variables other than icing for which theoretical prediction is not possible or for which the developmental data sets are semi-quantitative or noisy as in this study.

Section 2 describes the dataset, section 3 discusses the selection of a physically based predictor for potential icing rate, and section 4 outlines the statistical model. Section 5 presents the data analysis and section 6 discusses implementation of the algorithm.

## 2. Dataset

The observation set consists primarily of fishing vessels, fish processors, tow boats and Coast Guard vessels, which operate in Alaskan waters. A total of 195 icing incidents for December 1979 to December 1983 (Fig. 2) were obtained from the radio log of WBH29, a private reporting station on Kodiak Island. Most vessels ranged in length from 20–75 m. For fishing vessels, the vessel operator who made the report was interviewed to determine the extent of the icing event and provide additional meteorological information and vessel characteristics. This was difficult for the 1979–81 reports since it was often impossible to locate the operator. For tow boats, the operating logs of individual vessels were consulted. Tow boats were a particularly good source of data because their schedules were not affected by the fishing season or bad weather. United States Coast Guard and NOAA vessel reports were consise and usually required little follow up. A final consistency check of the data was made by comparing the individual reports with National Weather Service (NWS) sea level pressure analyses and air temperature fields.

A total of 85 verified reports were obtained (Table 2) and are tabulated in Pease and Comiskey (1985). Reports include date, location, ship's heading and speed, wind velocity, air temperature, sea height, maximum icing rate, total accumulation of ice and duration of the icing event. For each report, surface wind, sea height, and air and dewpoint temperatures were extracted from NWS Anchorage Forecast Office analyses. Sea surface temperatures (SST) were obtained from

TABLE 1. Comparison of icing rate nomenclature.

| | Mertins (1968) (cm/24 h) | Kachurin et al.* (1974) (cm/h) | Sawada (1973) (cm/h) | Lundqvist and Udin (1977) | This report (cm/h) |
|---|---|---|---|---|---|
| Light | 1–3 | — | <0.5 | 0.5–2 cm/12 h | <0.7 |
| Moderate | 4–6 | — | 0.5–2 | 1–3 cm/4 h | 0.7–2.0 |
| Heavy or severe | 7–14 | 1.8 | >2 | >4 cm/4 h | >2.0 |
| Very severe | >15 | 4.2 | — | — | — |

* Using the Kachurin et al. (1974) nomogram to convert from tons $h^{-1}$ to cm $h^{-1}$.

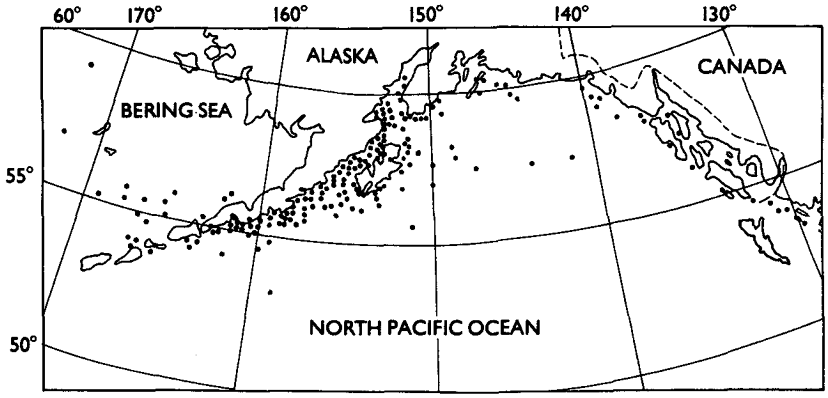


FIG. 2. Superstructure icing incidents from December 1979 to December 1983 from the radio log of WBH29 (Peggy Dyson).

NOAA regional sea surface temperature products. The data were examined for consistency between the vessel observations, the NOAA analyses, and the narrative of the vessel operators. In three cases the wind speeds were increased and in six cases the SST were adjusted based upon the narrative. The air temperatures from the NWS-analyzed charts were more regionally consistent than those estimated by the vessel operators. Many vessels have uncalibrated thermometers or no thermometers and there were frequent errors in guessing the temperature by as much as 5°C. One-third of the ship observations deviated from the NWS analysis by greater than 2°C. We have used hand-analyzed NWS air temperature fields from the Alaskan Region for the algorithm development.

Most captains have good facility with the Beaufort wind scale. In several extreme cases along mountainous coasts wind estimates from the NWS surface analysis were half the observed winds. The only persistent errors in using the captains' winds occurred in moderate wind conditions when they appeared to underestimate wind speed. The maximum reported wind, either from the captain or the NWS analysis, is used as the closest approximation to the regional wind at the time of observation.

It is necessary to define "potential icing rate" in terms of the data set. As previously indicated, potential icing is defined as a sustained icing rate by a vessel that is not actively avoiding icing through heading downwind, moving at slow speeds, or avoiding open seas. Total accumulation of ice divided by duration of the event may systematically underestimate icing rates since the vessel is often transiting different conditions; this could happen even though event durations were typically less than 12 h. Furthermore, maximum observed rates have scatter and do not necessarily represent sustained rates. A good compromise is to define potential icing as the average of the maximum observed rate and the total event rate. In most cases the two rates were similar (Pease and Comiskey, 1985).

TABLE 2. Icing reports relative to vessel length ($l$) in meters for 27 vessels and 84 observations (excluding 1 report from the drill rig *Ocean Bounty*).

| $l < 35$ | $35 \leq l < 55$ | $55 \leq l < 75$ | $75 \leq l$ |
|---|---|---|---|
| | Number of vessels | | |
| 12 | 6 | 6 | 3 |
| | Number of observations | | |
| 18 | 46 | 16 | 4 |
| | Names of vessels (number of observations) | | |
| Crabber *Alaska Trojan* (3)<br>USCGC *Cape Coral* (3)<br>Crabber *Hermitage* (2)<br>Tug *Sandra Foss* (2)<br>(eight other crabbers, shrimpers, bottom fishers, USCGC) | Tug *Justine Foss* (18)<br>Tug *Crusader* (9)<br>Coastal freighter *Snow Bird* (8)<br>Tug *Leslie Foss* (6)<br>NOAA RV *Chapman* (3)<br>Conv. Army F.S. *Princess Tamara* (2) | NOAA RV *Miller Freeman* (6)<br>USCGC *Sedge* (4)<br>USCGC *Storis* (3)<br>USCGC *Confidence* (1)<br>USCGC *Sweetbrier* (1)<br>USCGC *Planetree* (1) | Ferry *Tustamena* (2)<br>USCGC *Boutwell* (1)<br>NOAA RV *Surveyor* (1) |

The ship's heading relative to the wind direction showed that most cases with anomalously low icing rates for the meteorological conditions were from vessels taking less spray because they were heading downwind compared to vessels with wind abeam or heading into the wind. We used the criterion of a heading of 120° or greater off the wind, or of a captain's comment in the narrative reports that he was running downwind, to identify these cases. There are 15 cases meeting the downwind criterion. Additional low icing cases relative to meteorological conditions were identified for vessels taking less spray because the sea heights were anomalously low in the lee of an island or in an otherwise undeveloped sea. We used the criterion of wave height to wind speed ratio of less than 0.15 m to 1.0 m $s^{-1}$ to indicate sheltered conditions; there were 12 nondownwind cases and 7 downwind cases in this category. We retained one extreme icing case that met the low wave height criteria but exhibited no other suggestion of lee shore modification. Of the 85 cases, 58 are considered meteorologically consistent, nondownwind, open-ocean cases of potential icing rate.

## 3. Meteorological parameters associated with marine ice accretion

Physical processes associated with ice accretion are numerous and complex. They are traditionally divided into two general parts in theoretical analyses: the efficiency with which water is delivered to the substrate (Zakrzewski, 1986), and thermodynamic analysis of the freezing on the icing surface (List, 1977; Ackley and Templeton, 1979; Lozowski and Gates, 1985). Theoretical modeling of the rate of atmospheric icing on wires has been very successful (Lozowski et al., 1983; Makkonen, 1984). Atmospheric icing considers small airborne drop sizes (~45 $\mu$m), the fluid dynamics of flow around a cylinder, and heat balance between substrate/ice/water/air interfaces. Recent results suggest that the dependence of the atmospheric icing rate on free-stream wind speed is greater on rough surfaces than on smooth surfaces (Makkonen and Stallabrass, 1985). Two models which consider vessel icing (Kachurin et al., 1974; Stallabrass, 1980) are based on modifications of techniques of modeling atmospheric icing. They have generally been less successful compared to modeling of stationary wires because of the difficulty in formulating the vessel icing problem (Kachurin et al., 1974; Jessup, 1985). Given an adequate supply of water to the surface, models show that the most important factor in determining icing rate is the sensible heat flux from the icing surface to the airstream, which is a function of wind speed and the difference between the air temperature and the freezing point (Stallabrass, 1980; Lozowski and Gates, 1985).

The primary water source for hazardous ice accretion on vessels is wave-generated spray produced by the impact of waves against the vessel hull (Borisenkov and Panov, 1972; Shellard, 1974; Zakrzewski, 1986). The frequency of impact depends on the speed and direction of the vessel relative to the wave length and direction of the wave field. The heading of the ship with respect to the wind has a pronounced effect on the rate at which ice accumulates on the vessel. A vessel running with the wind in freezing conditions will be iced up much less than one steaming against it (Mertins, 1968; Minsk, 1977; Panov, 1978; Pease and Comiskey, 1985). In gale force winds the amount of free water driven over the vessel can be very considerable (Hay, 1956). Stallabrass (1980) notes the use of drop size distribution to estimate water supply on vessels as used in atmospheric icing is inappropriate because the water is encountered in the form of "gushes" and "cascades." Zakrzewski (1986) has estimated that a rate of 50 kg $m^{-2}$ $min^{-1}$ for a vessel steaming into a 20 m $s^{-1}$ wind is not unreasonable. He also notes that the volume of water available for freezing is ten times greater for a wind speed of 20 m $s^{-1}$ than 10 m $s^{-1}$. Some authors report minimum threshold wave heights for icing and severe icing of 1.5 and 2.5 m, respectively, due to lack of available water source (Sawada, 1966, 1973). A wave height of 2.5 m approximately corresponds to a wind speed of 10 m $s^{-1}$ over 10 h. Clearly the rate of water supply by waves is important for vessel icing, particularly as a limiting factor for low wind speeds. Also important is the rate of runoff from the vessel, but none of the processes associated with runoff are completely simulated in any of the existing models (Jessup, 1985). In a calculation of a severe icing case near Iceland, Hay (1958) assumed that 10% of the incident spray froze on the vessel. Kachurin et al. (1974) assumed that 25% froze.

The influence of SST on icing rate is more problematic than the influence of wind speed and air temperature. Extreme icing, associated with vessel loss, often occurs with water temperatures below 2°C (Lee, 1958; George, 1975). Rapid accretion has been reported in the vicinity of the ice edge when the water temperature was below 2°C in the Barents Sea (Lee, 1958), Greenland Sea (De Angelis, 1974), and Sea of Okhotsk (Sawada, 1966). Mertins (1968) and Lundqvist and Udin (1977) show decreasing icing rate with increasing SST up to 6°C. A common practice to avoid icing conditions in the Gulf of Alaska is for vessels to run for warm sea temperatures further offshore. One Alaskan report stated that a method to minimize icing, although not recommended, is to increase the amount of warm seawater running across the deck. Shellard (1974) notes that icing is still possible at a water temperature of 6°C if atmospheric conditions are extreme, and that SST is probably an important determiner of icing rate.

In the vessel icing models of Kachurin et al. (1974) and Stallabrass (1980), the influence of sea temperature is as a heat source since the air must cool the water temperature to the freezing point before the spray can freeze. In their models this heat source effect is generally small compared to the latent heat of freezing. However, Jessup (1985) points out that other physical processes associated with SST may be important to vessel icing, because the colder the water, the better its chance of freezing before runoff. The sensitivity of icing rate to SST suggested by most subjective analyses of severe icing is greater than that of the models of Kachurin et al. (1974) and Stallabrass (1980).

Given the availability of water on vessel surfaces, the maximum rate of accretion is determined by the thermodynamic balance of ice growth and heat transfer away from the surface. Heat is carried away from a water surface by convection of sensible heat, evaporative heat flux, and radiative cooling. These are balanced by the cooling of the seawater to the freezing point, runoff from the vessel of a portion of the cooled water, and the production of ice through freezing. There are secondary, vessel-dependent fluxes which are not considered here such as the heat flux between the ice and the underlying surface (Jessup, 1985).

The primary thermodynamic balance at the air–ice interface per unit surface area is

$$Q_{\mathrm{lat}}+Q_{\mathrm{im}}+Q_{\mathrm{ro}}=Q_{\mathrm{conv}}+Q_{\mathrm{evap}}+Q_{\mathrm{rad}} \quad (1)$$

where

| | |
|---|---|
| $Q_{\mathrm{lat}}$ | the latent heat flux released at the outer ice surface due to freezing |
| $Q_{\mathrm{im}}$ | heat flux in cooling the seawater from the sea temperature to the freezing point for the water which remains accreted to the surface |
| $Q_{\mathrm{ro}}$ | heat flux in cooling the runoff water to the temperature it had when leaving the surface |
| $Q_{\mathrm{conv}}$ | heat flux from the outer surface into the airstream caused by forced convective (sensible) heat transfer |
| $Q_{\mathrm{evap}}$ | evaporative heat flux from the outer surface to the airstream |
| $Q_{\mathrm{rad}}$ | energy flux due to radiative transfer |

This primary balance per unit surface area (see Jessup, 1985) can be approximated as

$$L_i\rho_i\frac{dH_i}{dt}+F\rho_w\frac{dH_w}{dt}C_w(T_w-T_f)+(1-F)\rho_w\frac{dH_w}{dt}C_w(T_w-\mathrm{T_{ro}})=C_H\rho_aC_aV_a[(T_f-T_a)+\eta(e_s-0.9e_a)]+\sigma(T_f^4-\epsilon_aT_a^4) \quad (2)$$

where

| | |
|---|---|
| $\rho_i$, $\rho_w$, $\rho_a$ | density of saline ice, seawater and air, respectively |
| $T_f$, $T_w$, $T_a$ | temperature of saline ice at the freezing point, seawater and air, respectively |
| $T_{\mathrm{ro}}$ | average temperature of runoff leaving the vessel |
| $F$ | fraction of impinging seawater remaining on the vessel and available for freezing; it is probably a function of waveheight and freezing rate |
| $L_i$ | latent heat of freezing of saline water |
| $C_w$, $C_a$ | specific heat of seawater and dry air, respectively |
| $\frac{dH_i}{dt}$ | rate of ice formation |
| $\frac{dH_w}{dt}$ | rate of impingement of seawater |
| $C_H$ | transfer coefficient of heat flux |
| $V_a$ | wind speed |
| $e_s$, $e_a$ | vapor pressure of saturated and ambient air, respectively |
| $\eta$ | a constant approximately equal to 16°C $\mathrm{kP}_a^{-1}$ |
| $\sigma$ | Stefan–Boltzmann constant |
| $\epsilon_a$ | emissivity of the air with values ~0.5 to 1.0; higher values are for fog and overcast |

Simplifying assumptions have been made in calculating the sensible heat flux in addition to assuming a steady-state freezing process. One is that the difference between the ambient air temperature and freezing point is used, not an intermediate value between $T_f$ and $T_w$. The other is that the temperature of impinging water is near $T_w$. We use a linear dependence of sensible heat flux on wind velocity, which is more appropriate for very turbulent boundary layers (Kraus, 1972) in contrast to a square root dependence, which is appropriate to smooth flow around pipes (Makkonen, 1981).

The water on the deck must be at the freezing point before icing can occur; therefore,

$$T_{\mathrm{ro}}\approx T_f. \quad (3)$$

This implies that $\rho_w dH_w/dt$ represents the water that freezes or forms runoff but excludes any immediate splash back. Using (3) and noting that

$$\rho_i\frac{dH_i}{dt}=\rho_wF\frac{dH_w}{dt} \quad (4)$$

where $F$ can also be interpreted as a freezing fraction, (2) becomes

$$\frac{dH_i}{dt}=\left\{\frac{C_H\rho_aC_aV_a}{\rho_iL_i}[(T_f-T_a)+\eta(e_s-0.9e_a)]+\frac{Q_{\mathrm{rad}}}{\rho_iL_i}\right\}\times\left[1+\frac{C_w}{L_iF}(T_w-T_f)\right]^{-1}. \quad (5a)$$

Because sensible heat flux is the primary determinant of vessel icing, the basic functional form for the dependence of icing rate on meteorological variables is

$$\frac{dH_i}{dt}\alpha\frac{V_a(T_f-T_a)}{1+\Phi(T_w-T_f)} \quad (5b)$$

where $\Phi\equiv C_w/L_iF$ and the proportionality is approximately $C_H\rho_aC_a/\rho_iL_i$. Section 5 will use (5b) as an index for potential icing in which the observation set will determine the numerical value of (5b) for the transition points between icing categories. The data will also be used to empirically determine a value of $\Phi$ which best

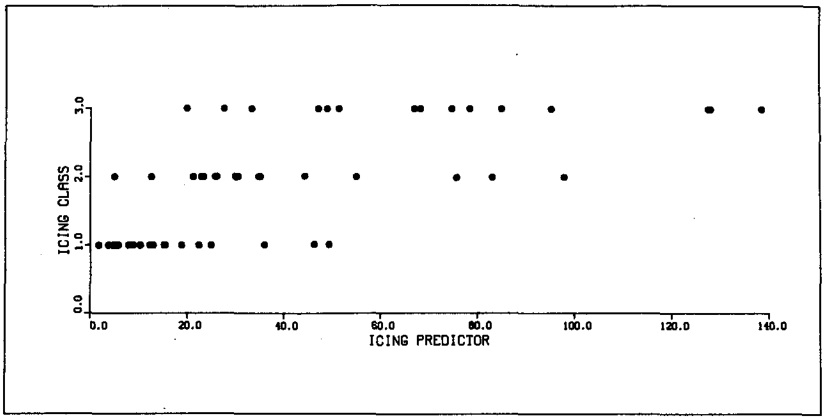


FIG. 3. Plot of 52 open-ocean, nondownwind bivariate observations. The abscissa is the predictor equation (5b) in units of m °C $s^{-1}$ with $\phi = 0.4$. The ordinate is icing class, light = 1, moderate = 2, heavy = 3.

models the mitigating effect of warm sea temperatures. Secondary variables which influence icing rate, such as wave height, relative humidity, and radiation, are assumed to be properties of the typical meteorological conditions associated with icing in Alaskan waters for moderate size vessels; although they contribute to icing, they do not vary independently from the primary meteorological variables sufficiently to provide additional predictive skill. Review of the literature on vessel icing does not suggest that icing in Alaskan waters is inherently different than in other geographical regions and suggest that (5b) represents the primary physics of the vessel icing process.

## 4. Statistical model

Since most observations of icing and associated meteorological parameters are approximate and sometimes subjective, classical procedures of data analysis such as regression techniques are inadequate for statistical models of vessel icing. The dataset and its errors do not necessarily satisfy Gaussian distribution assumptions. For example, one of our icing cases had an accumulation rate twice the rate which would have been predicted for the meteorological conditions based upon previous nomograms or other cases in our dataset. There is no doubt, however, that this was a severe case of icing, albeit probably overstated. It would be excluded as an outlying point from a traditional regression analysis even though the vessel meteorological data are consistent with the NWS analysis.

Our algorithm development is based upon the following procedures:

1) The predictor index (5b) is selected based upon physical insight.

2) The predictand icing rate is established a priori into fixed icing categories based upon the requirements of marine forecasting. These categories were selected as follows: light: less than 0.7 cm $h^{-1}$; moderate: between 0.7–2.0 cm $h^{-1}$; and heavy: greater than 2.0 cm $h^{-1}$ (Table 1). These break points roughly divide the data, and the categories compare favorably to the subjective nomenclature used by the Alaskan observers.

3) The "best" predictor is selected by varying $\Phi$, the

TABLE 3. Value of the absolute potential predictability (APP) as a function of the sea surface temperature (SST) coefficient $\Phi$ for different predictor class interval assumptions *m*. The APP is the value of the potential predictability (PP) as given in the Appendix minus that which could be obtained by chance ($\mathrm{PP}_{901}$), for the same sample size and value of *m*.

| *m* | APP | | | | | | | | | | | | $\mathrm{PP}_{901}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 9 | 0.194 | 0.174 | 0.174 | 0.174 | 0.174 | 0.194 | 0.194 | 0.174 | 0.174 | 0.174 | 0.155 | 0.146 | 0.241 |
| 8 | 1.162 | 0.133 | 0.103 | 0.141 | 0.141 | 0.141 | 0.145 | 0.145 | 0.145 | 0.224 | 0.229 | 0.166 | 0.216 |
| 7 | 0.171 | 0.122 | 0.122 | 0.138 | 0.138 | 0.138 | 0.146 | 0.146 | 0.146 | 0.146 | 0.195 | 0.159 | 0.195 |
| 6 | 0.216 | 0.216 | 0.216 | 0.235 | 0.235 | 0.235 | 0.235 | 0.203 | 0.203 | 0.203 | 0.167 | 0.155 | 0.170 |
| 5 | 0.179 | 0.167 | 0.179 | 0.132 | 0.172 | 0.172 | 0.172 | 0.172 | 0.172 | 0.172 | 0.172 | 0.235 | 0.152 |
| 4 | 0.186 | 0.146 | 0.146 | 0.146 | 0.146 | 0.146 | 0.146 | 0.146 | 0.146 | 0.146 | 0.146 | 0.146 | 0.126 |
| 3 | 0.252 | 0.252 | 0.252 | 0.269 | 0.269 | 0.269 | 0.246 | 0.252 | 0.231 | 0.252 | 0.252 | 0.231 | 0.101 |
| SST coefficient | 0.01 | 0.05 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.2 | |

TABLE 4. Contingency table of bivariate data for $m = 6$ and $\Phi = 0.4$.

| Icing category | Predictor interval 1 | 2 | 3 | 4 | 5 | 6 | Sum |
|---|---|---|---|---|---|---|---|
| 3. Heavy | 0 | 0 | 2 | 1 | 6 | 6 | 15 |
| 2. Moderate | 1 | 1 | 5 | 7 | 1 | 3 | 18 |
| 1. Light | 7 | 7 | 2 | 1 | 2 | 0 | 19 |
| Sum | 8 | 8 | 9 | 9 | 9 | 9 | 52 |

TABLE 6. The value of the $a_s$ score as a function of predictor class intervals $m$ for $\Phi = 0.4$. The $a_{901}$ is the value obtained from random data for the same sample size and $m$ value (Appendix).

| $m$ | Predictor interval 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|
| $a_0$ | 0.731 | 0.615 | 0.654 | 0.731 | 0.673 | 0.673 | 0.731 |
| $a_{901}$ | 0.500 | 0.519 | 0.538 | 0.558 | 0.577 | 0.577 | 0.596 |
| $a_s$ | 0.231 | 0.096 | 0.116 | 0.173 | 0.096 | 0.096 | 0.135 |

sea temperature coefficient, until the absolute potential predictability (APP, defined in the Appendix) for the dataset is a maximum.

4) The observations are divided into a specified number of equally populous class intervals for the predictor. Let the number of observations be $a_{ij}$ for the $i$th predictor class, $i = 1, m$, where $m$ is arbitrary, and the $j$th icing class, $j = 1, n$; $n = 3$. An $m \times n$ contingency table is developed, for which every predictor class $i$ is assigned to icing category $j$ such that the conditional probability of icing given a predictor class will be a maximum:

$$P(j|i) = a_{ij}/a_i$$

where

$$a_i = \sum_{j=1}^{n} a_{ij}. \tag{6}$$

The sum of the number of correctly assigned forecast/observed entries in the contingency table divided by the total number of observations, called the "$a_0$ score," provides a measure of forecast skill of the candidate algorithm. Note that the $a_0$ score will not necessarily be zero even if the icing categories were assigned to the predictors at random. The number of class intervals, $m$, for algorithm is selected by maximizing

$$a_s = a_0 - a_{901} \tag{7}$$

where $a_{901}$ is the 901st of a sequence of ascending values of the $a_0$ statistic computed from 1000 applications of the procedure for random data with the same sample size and value of $m$ as the observational set. Further details are provided in the Appendix.

5) An independent estimate of $a_0$ is computed from a withheld subset of observations. Ten percent of the total number of cases are saved as an independent check on the statistical procedure.

With this procedure a methodology is established to select the compromise between a small number of predictor classes with many observations in each and a large number of classes with few observations, each of which is sensitive to inaccuracies in basic data set.

## 5. Data analysis

Of the 58 open-ocean, nondownwind cases, six (Nos. 5, 15, 25, . . . , from Table 2; Pease and Comiskey, 1985) were withheld as an independent set. The remaining 52 bivariate pairs are shown in Fig. 3.

The first task was to determine the final form of the predictor variable by determining the value of the SST coefficient $\Phi$. This was done by maximizing the APP (See Appendix). The APP is a good measure of the predictor's capability to separate the data into predictand categories. Predictor variables other than $\Phi$ could be evaluated at this step. Table 3 lists the value of APP as a function of $\Phi$ for a sequence of predictor class intervals $m$. For example, with $m = 6$ and $\Phi = 0.4$ the data are grouped as in Table 4 with APP = 0.235. The maximum in Table 3 is for $\Phi = 0.2$–$0.4$ with $m = 3$. Other maximums are $\Phi = 0.2$–$0.5$ with $m = 6$ and $\Phi = 0.4$–$0.5$ with $m = 9$. Also note the lack of sensitivity for $m = 4$ and $m = 7$. The technique shows that $\Phi = 0.2$–$0.5$ is a reasonable range. Based on APP value, we established predictor (5b) with $\Phi = 0.4$.

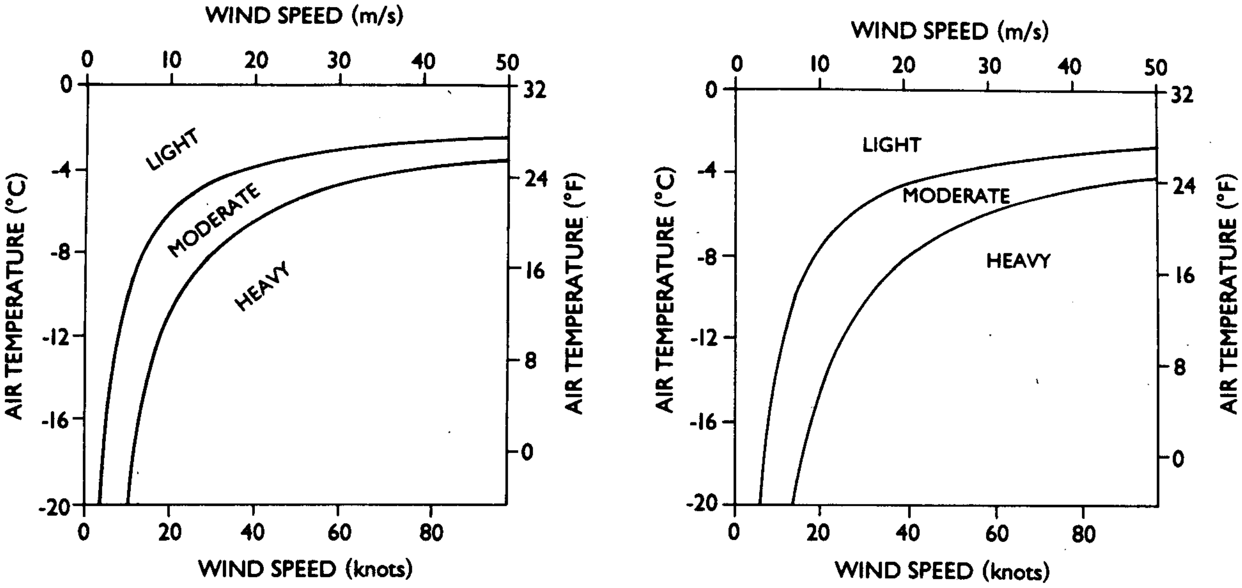
We now assigned each predictor interval ($i = 1, m$; $m = 3$ to 9) to an icing category $j = 1, 2$ or 3 such that the conditional probability of icing given the predictor is a maximum. For the contingency table with $m = 6$ (Table 4), predictor intervals 1 and 2 are assigned to light icing; 3 and 4 to moderate; and 5 and 6 to heavy. From this assignment a forecast/observed table was formed (Table 5). The $a_0$ score for this case ($m = 6$, $\Phi$

TABLE 5. Forecast/observed table for $m = 6$ and $\Phi = 0.4$ for icing classes 1, 2, and 3.

| Observed category | Forecast category 1 | 2 | 3 |
|---|---|---|---|
| 3. Heavy | 0 | 3 | 12 |
| 2. Moderate | 2 | 12 | 4 |
| 1. Light | 14 | 3 | 2 |

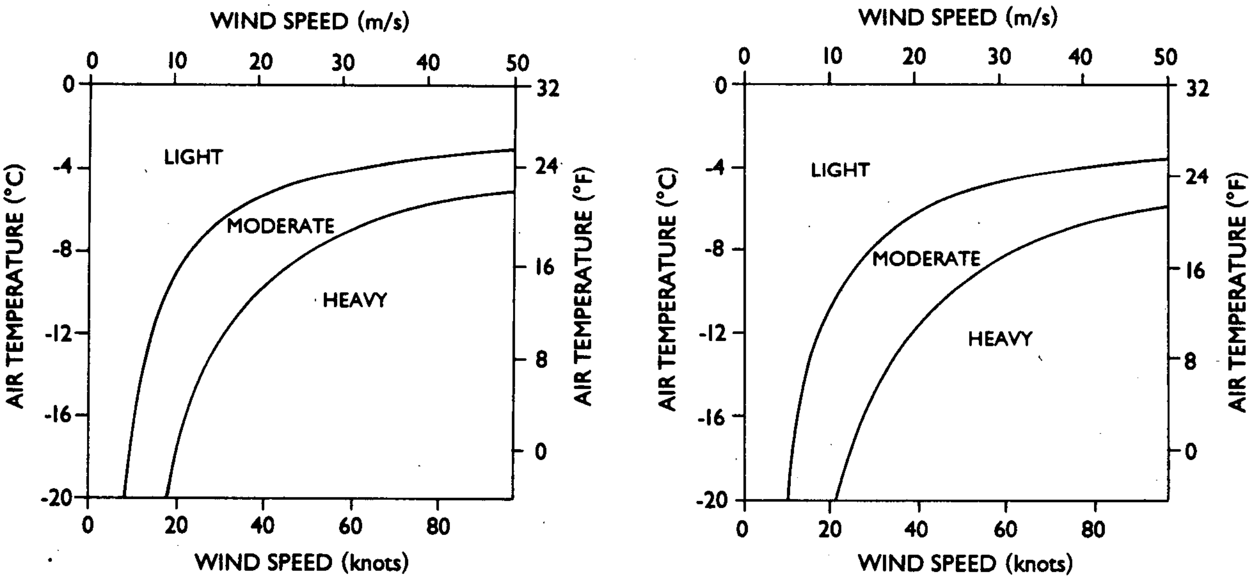
TABLE 7. Forecast/observed table for $m = 3$ and $\Phi = 0.4$ for icing classes 1, 2, and 3.

| Observed | Forecast 1 | 2 | 3 |
|---|---|---|---|
| 3. Heavy | 1 | 2 | 12 |
| 2. Moderate | 2 | 12 | 4 |
| 1. Light | 14 | 3 | 2 |



Icing conditions for vessels heading into or abeam of the wind for water temperatures of + 1°C (34°F)

Icing conditions for vessels heading into or abeam of the wind for water temperatures of +3°C (37°F)



Icing conditions for vessels heading into or abeam of the wind for water temperatures of +5°C (41°F)

Icing conditions for vessels heading into or abeam of the wind for water temperatures of +7°C (45°F)

**Light Icing - Less than 0.7cm/hr (0.3in/hr)**
**Moderate Icing - 0.7cm/hr (0.3in/hr) to 2.0 cm/hr (0.8in/hr)**
**Heavy Icing - Greater than 2.0cm/hr (0.8in/hr)**

FIG. 4. Icing nomograms for four values of sea temperature.

TABLE 8. Forecast/observed table for independent data. The $a_0$ score was 0.833.

| | Forecast | | |
|---|---|---|---|
| Observed | 1 | 2 | 3 |
| 3. Heavy | | | 4 |
| 2. Moderate | | | 1 |
| 1. Light | 1 | | |

= 0.4) is the number of correct forecasts (38) divided by the total (52) or 0.731. This means almost three-quarters of the observations were correctly placed and, given that some of the off-diagonal cases in Table 5 represent outlying data which are not part of the population, the score is reasonably good. Table 6 provides the $a_s$ score ($a_s = a_0 - a_{901}$) as a function of the number of predictor class intervals $m$. Also shown are the $a_{901}$ values. As suggested by the APP scores in Table 3, $m = 3$ and $m = 6$ were candidates with $m = 3$ having the highest $a_s$ score (Table 7). Both $m = 3$ and $m = 6$ break moderate/heavy icing at a predictor value of 45.2 (m °C s$^{-1}$); $m = 3$ breaks light/moderate at 20.6 (m °C s$^{-1}$); and $m = 6$ breaks light/moderate at 19.3 (m °C s$^{-1}$). We took the $m = 3$ model and forecast the icing class by the above range of predictor values.

The forecast/observed table for the six independent observations is given in Table 8. The $a_0$ value is 0.83 which is equal to the $a_{901}$ score of 0.83. All but one observation were correctly forecast, but, in practical terms, the test is barely significant because of the small sample size.

TABLE 9. Categorical forecast procedure.

| | Icing class | | |
|---|---|---|---|
| | Light | Moderate | Heavy |
| Icing rate (cm h$^{-1}$) | <0.7 | 0.7 to 2.0 | >2.0 |
| Predictor (PR)* (m °C/s) | <20.6 | 20.6–45.2 | >45.2 |

* $PR = V_a(T_f - T_a)[1 + 0.4(T_w - T_f)]^{-1}$

where

$V_a$ wind speed (m s$^{-1}$)
$T_f$ freezing point of seawater (−1.7°C for North Pacific)
$T_a$ air temperature (°C)
$T_w$ sea temperature (°C)

## 6. Forecast procedure

The categorical forecast procedure is summarized by Table 9 and Fig. 4. The value of the predictor is calculated as a function of wind speed, freezing point of seawater, and air and sea temperature. If the value of the predictor is less than 20.6 (m °C s$^{-1}$), icing is considered light (<0.7 cm h$^{-1}$). If the value of the predictor is greater than 45.2 (m °C s$^{-1}$), icing is considered heavy (>2.0 cm h$^{-1}$). Four plots to forecast icing categories as a function of wind speed and air temperature are given in Fig. 4 for four SST values.

The forecast procedure considers only the major features of the icing process. If the value of the predictor is near the transition between classes, the forecaster should consider the probable errors or bias in input parameters and the importance of secondary physical

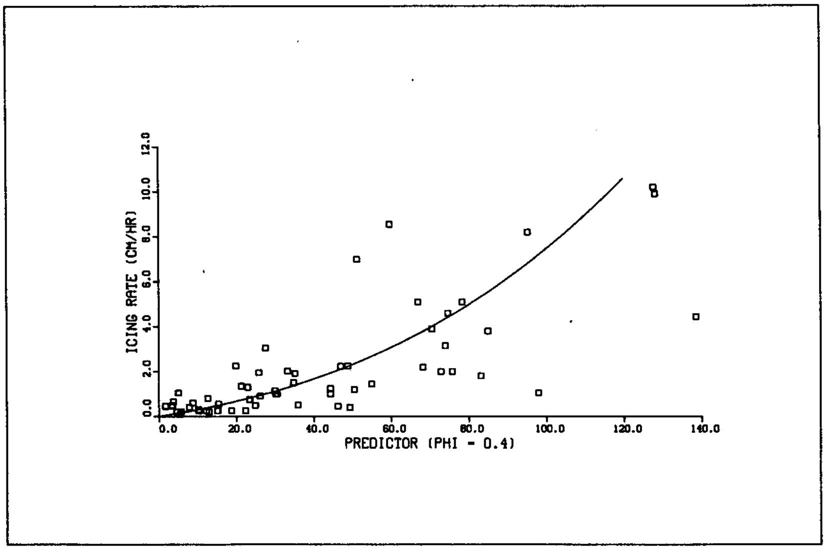


FIG. 5. Continuous algorithm. (See Table 10.) Data points are the 58 open-ocean, non-downwind icing cases.

TABLE 10. Icing rate algorithm for continuous variables.

| | Icing class | | | |
|---|---|---|---|---|
| Parameter | 0 | ½ | ⅔ | 3 |
| Icing rate (IR)* (cm h$^{-1}$) | 0.0 | 0.7 | 2.0 | 4.1 |
| Predictor (PR)† (m °C/s) | 0.0 | 20.6 | 45.2 | 71.4 |

* $\mathrm{IR}(\mathrm{cm\,h^{-1}}) = A(\mathrm{PR}) + B(\mathrm{PR})^2 + C(\mathrm{PR})^3$

where

$A = 2.73 \times 10^{-2}$
$B = 2.91 \times 10^{-4}$
$C = 1.84 \times 10^{-6}$

† $\mathrm{PR} = V_a(T_f - T_a)[1 + 0.4(T_w - T_f)]^{-1}$

where

$V_a$ wind speed (m s$^{-1}$)
$T_f$ freezing point of seawater (−1.7°C for North Pacific)
$T_a$ air temperature (°C)
$T_w$ sea temperature (°C)

processes. For example, if the air mass is particularly dry and clear, then latent heat and radiative transfer may be greater than assumed by the algorithm and the predictor value should be increased. On the other hand, if the wave field is less than fully developed with waves <2 *m* in a fast moving weather system with strong winds, the predictor value should be decreased.

For some applications it would be more convenient to have an icing algorithm for a continuously varying predictor. In this study, the break points between classes and the median value of the icing rate and the predictor for the heavy icing category were used to fit a polynomial equation between icing rate and the predictor (Table 10 and Fig. 5). For a fixed air and sea temperature, Fig. 5 suggests that icing rate increases faster than linear with an increase in predictor value. This could imply that heat transfer is more efficient at higher wind speeds, that higher wave heights lead to more efficient transfer of water to exposed surfaces, or that increased ice salinity at fast freezing rates decreases the latent heat of freezing. The continuous formula in Table 10 does not imply more skill than the categorical algorithm in Table 9. In particular quantitative extrapolation of the predictor beyond a value of 70 (m °C s$^{-1}$) with an icing rate of 4.0 cm h$^{-1}$ is beyond the skill represented by our dataset. Our dataset did, however, have one verified icing case of 8 cm h$^{-1}$.

## 7. Discussion

With three predictand classes and a small number of predictor classes there is a trade-off between minimizing the influence of outlying data and sensitivity in determining predictor or model parameters such as $\Phi$ and *m*. This is particularly noticeable for the sufficient but relatively small sample size in this study since changing parameters shifts only a few bivariate pairs. Also, independent evaluation is difficult because there are so few observations. Complete evaluation must depend on further measurements.

It is interesting to compare the results from the statistical icing model to the theoretical form in (5). For example, $\Phi$ represents the first order dependence of icing on sea temperature:

$$\frac{C_w}{L_i F} = \Phi \approx 0.4\text{°C}^{-1}. \qquad (8)$$

Solving for the freezing fraction results in $F \approx 0.025$ or that 97% of the impinging water eventually runs off, given an adequate water supply and a sea temperature near our median value of 3°C. This can be compared to a typical freezing fraction of $F = 0.017$ calculated for an icing rate of 2 cm h$^{-1}$ and a water impingement rate on a vessel with 2.5 m freeboard (Zakrzewski, 1986) of 30 kg m$^{-2}$ min$^{-1}$. The parameter $\Phi$ is probably

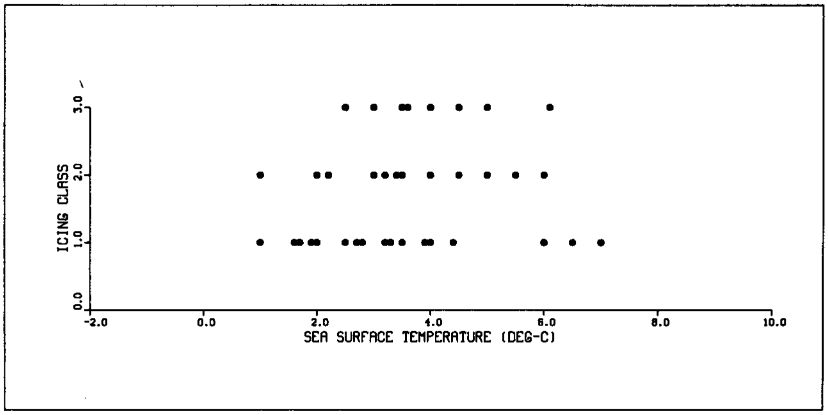


FIG. 6. Distribution of sea surface temperature for 58 observations. Several points are plotted at the same values. Note that this figure does not consider the severity of the meteorological conditions.

TABLE 11. Estimate of the vessel heat transfer coefficient $C_H$. Coefficient $A$ is from Table 10 and is based on cm h$^{-1}$.

$$\frac{C_H \rho_a C_a}{\rho_i L_i} \approx \frac{A}{1.3} \approx 20 \times 10^{-2}$$

$\rho_a = 1.35$ kg m$^{-3}$
$\rho_i = 0.90 \times 10^3$ kg m$^{-3}$
$C_a = 1004$ j kg$^{-1}$ °C$^{-1}$
$L_i = 2.3 \times 10^5$ j kg$^{-1}$ (salinity = 10 ppt)
(cm h$^{-1}$)(m s$^{-1}$)$^{-1}$ $= 3.6 \times 10^5$
$C_H \sim 9 \times 10^{-3}$

a function of sea temperature and decreases as the sea temperature approaches the freezing point since more water would probably freeze on deck before running off. However, the SST influence on reducing icing is small in this range. The $\Phi$ parameter can be thought of as the coefficient of the linear approximation to the SST influence on icing centered on 3°C (Fig. 6).

We can also estimate the heat transfer coefficient $C_H$ for a typical vessel (Table 11) by assuming that evaporative flux and other unmodeled physics represent 30% of the sensible heat flux value and by equating the leading constants in (5) to the coefficient of the predictor in the linear term of the polynomial given in Table 10. A value of $C_H \sim 9 \times 10^{-3}$, while speculative, is consistent with an effective value of heat flux as 10% of the drag value quoted for vessels (Owen and Thomson, 1963; Hoerner, 1965) and heat transfer coefficients for buildings (Threlkeld, 1970).

There are 15 verified reports of icing rates greater than 2.0 cm h$^{-1}$ from our Alaskan dataset. Our rates and range of meteorological conditions for icing compare to the Soviet literature, which had the majority of cases from the Bering, Okhotsk, and Barents Seas (Kachurin et al., 1974; Panov, 1978; Fig. 7), and Lundqvist (1977) for the Baltic Sea. These results contrast with the nomogram from Mertins (1968) in which the maximum icing rate given is 15 cm per 24 h. A comparison of various standard cases is shown in Table 12. Similar differences are found by Brown and Agnew (1985b). We believe that the principal causes of icing are geographically invariant and that conditions in different regions cannot account for the difference between the Mertins (1968) and the Stallabrass (1980) methods, and the Kachurin et al. (1974) and PMEL datasets and nomograms.

We assume that low predicted values of icing from some methods are due, in part, to the use of small datasets which did not have icing rates greater than 2.0 cm h$^{-1}$. For example Stallabrass (1980) presents three maximum cases between 1.0 and 1.3 cm h$^{-1}$ from a set of 32 nondownwind observations from trawlers. Stallabrass (1980) suspected that his limited dataset contained "bad" points (by being too low) and had to rely on data provided by untrained observers who made their observations under circumstances that were far from ideal (Jessup, 1985). The Alaska dataset of 38 observations from Wise and Comiskey (1980) were from very mild winters (1977–1979) with no cases greater than 2.0 cm h$^{-1}$, and the Wise and Comiskey (1980) nomogram incorporated Mertins' nomogram. In contrast, De Angelis (1974) notes cases with extreme accumulation (>7 cm h$^{-1}$) and many authors quote rates in excess of 2.0 cm h$^{-1}$. We recommend that the Kachurin nomogram not be recalibrated for eastern Canadian waters using the Stallabrass dataset (Jarvis, 1983; Jessup,1985) despite real differences in typical air masses and ship design. Our experience—and the basis for this paper—is that icing data for the preparation of algorithm and nomograms cannot be collected routinely, but depend on a dedicated program with follow-up for each individual report.

The second possible contribution to low icing predictions is the use of inhomogeneous datasets. For example, Mertins (1968) used observations from a large dataset (400) which included a large range of vessel size and speed (Shellard, 1974). Icing rates for the same meteorological predictor can be substantially reduced by heading the vessel downwind or seeking windward shores despite high winds and low temperatures (Panov, 1978). Of the downwind cases in our study where the meteorological predictor value would have classified the icing rate as heavy, the median icing rate was 0.7 cm h$^{-1}$ in contrast to the median of the open-ocean, nondownwind cases of 4.1 cm h$^{-1}$. Minsk (1977) notes

TABLE 12. Comparison of icing rates. Rates are given in cm h$^{-1}$. All values were obtained from the nomograms of the authors except for this study which used the formula in Table 10. Similar differences between the Wise and Comiskey (1980) and Kachurin et al. (1974) models were noted by Brown and Agnew (1985b).

| $T_w$ (°C) | $T_f$ (°C) | $T_a$ (°C) | $V_a$ (m s$^{-1}$) | Mertins (1968) | Wise and Comiskey (1980) | Stallabrass (1980) | Kachurin et al. (1974) | This study | Lundqvist and Udin (1977) | De Angelis (1974) |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | −1.7 | −10. | 20.6 | 0.6 | 0.7 | 0.9 | 5.4 | 7.7 | | |
| 2.0 | −1.8 | −12. | 18.0 | 0.5 | 0.5 | 1.1 | 5.2 | 3.5 | | |
| 0.0? | −1.8 | −9. | 15.4 | 0.3 | 0.6 | 0.7 | 3.2 | 3.4 | | 7.6 |
| 3.0 | −1.7 | −10. | 20.6 | 0.4 | 0.5 | 0.8 | 4.7 | 3.0 | | |
| 2.0 | −1.8 | −12. | 12.0 | 0.3 | 0.4 | 0.7 | 2.9 | 2.2 | | |
| 1.0? | −0.5 | −6. | 10.0 | | | | | 1.3 | 1.0 | |
| 2.0 | −1.7 | −8. | 7.0 | 0.0 | 0.0 | 0.3 | 1.0 | 0.6 | | |

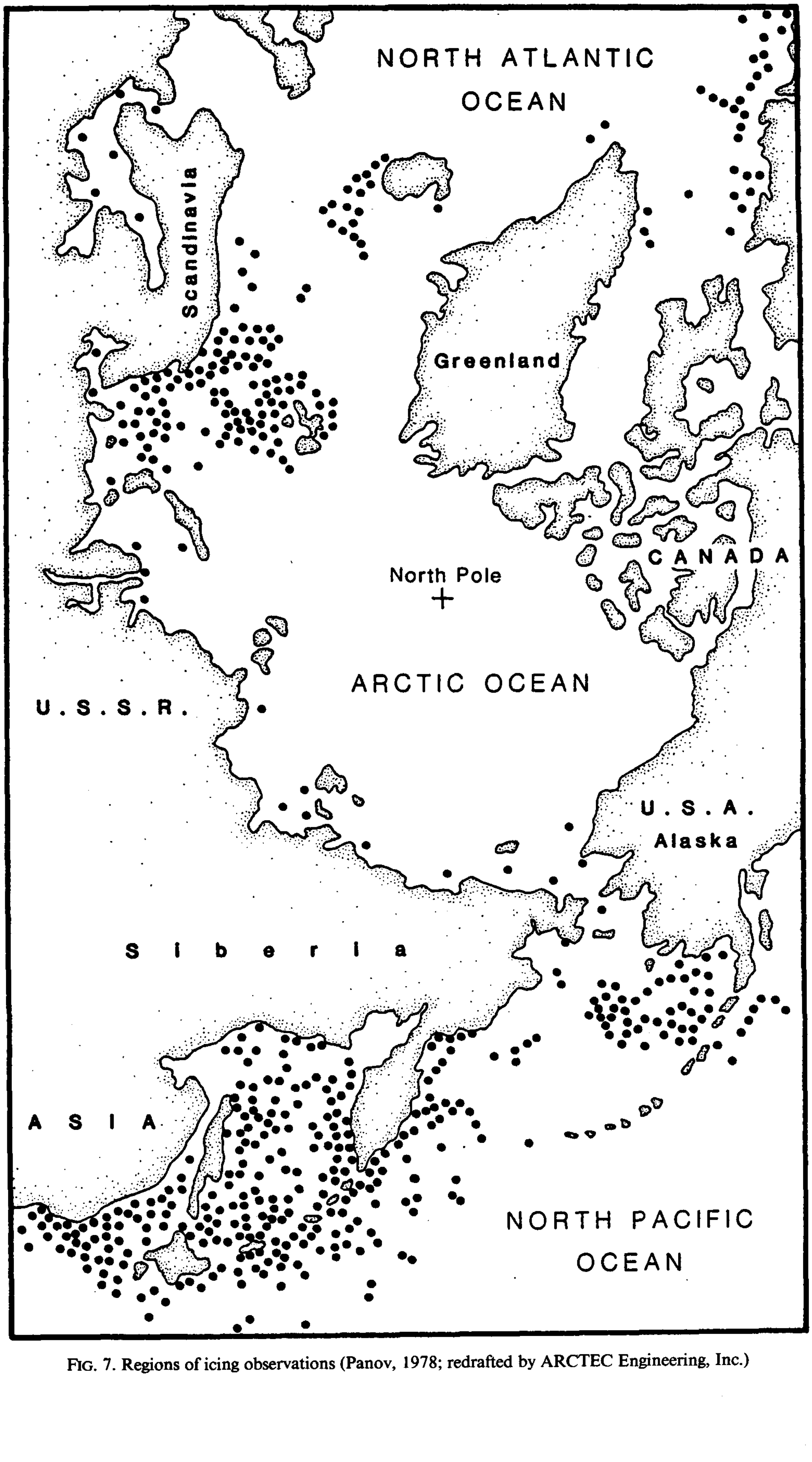


FIG. 7. Regions of icing observations (Panov, 1978; redrafted by ARCTEC Engineering, Inc.)

little or no accumulation even on the ship's stern when vessels are headed downwind, which is similar to our results. It is also not clear whether there would be an even distribution of upwind and downwind icing reports in extreme conditions. Differences in the definition of duration may be important. The use of rate categories per 24 h by Mertins (1968) may have contributed to overestimation of the event duration and underestimation of icing rate. Therefore, we are particularly concerned with contamination by downwind observations, overestimation of duration, and observations from large vessels or platforms.

George (1975) and, later, Stallabrass (1980) state that Mertins (1968) and thus Wise and Comiskey (1980) underpredict icing rate for a given meteorological condition. We confirm this result and emphasize the importance of open-ocean, nondownwind, medium size vessel observations in the definition of potential icing rate. While recognizing that there are some geographical differences in air mass and typical vessel design, we note there are no major differences in the principal causes of icing between regions. We recommend that the categorical icing algorithm given in this paper replace the Mertins (1968) and Wise and Comiskey (1980) nomogram. The decision is based upon our new observational dataset and comparison with the Soviet, Japanese, and Swedish reports (Table 12). The number of observations in our dataset supports the skill of the categorical algorithm. A strong statement is that a value of the predictor greater than 45 (m °C s$^{-1}$) corresponds to heavy icing (>2.0 cm h$^{-1}$).

## 8. Conclusion

Vessel icing is an extreme hazard to expanded operations in northern waters and generally occurs during episodes of strong cold air advection. The difficulty of taking icing observations, and the real variations between vessels produce large scatter in the data and forecasting methods, despite the best efforts of observers and analysts. The application of robust analysis techniques to a new dataset has produced a categorical algorithm that relates icing rate to meteorological variables. It is consistent with a large number of subjective and observed icing rates from different icing regions. We recommended that the Wise and Comiskey (1980) and Mertins (1968) nomograms be replaced with the new categorical icing algorithm. Results suggest that vessel icing can be predicted from the PMEL algorithm (Table 9) with some skill and that improved forecast services can be provided to vessels which are at risk in high latitude.

*Acknowledgments.* This paper is a contribution to the Marine Services Project at PMEL. We appreciate the assistance of the mariners who provided the observations and Peggy Dyson of WBH29 in Kodiak. The data were collected by the staff of the Arctic Environmental Information and Data Center of the University of Alaska under contract to NOAA's Pacific Marine Environmental Laboratory in Seattle, Washington. L. Leslie, AEIDC, Anchorage, and R. Brown and S. Galt, PMEL, contributed to the data processing. L. Leslie suggested the wave height to wind speed ratio for indicating sheltered conditions. We appreciate the discussions with D. Feit, K. A. MacDonald, and other participants at the Workshop on Offshore Winds and Icing, Halifax, N.S., 9–11 October 1985, and also with the many members of the National Weather Service who are committed to improved safety at sea.

## APPENDIX

### Computations

The statistical model is based on a conditional probability approach first suggested by Preisendorfer for visibility variables (Preisendorfer, 1983; unpublished manuscript). Let the number of observations be $a_{ij}$ for the $i$th predictor class, $i = 1, m$, and the $j$th icing class, $j = 1, n$. From a training subset of the observed data, estimate the conditional probability of icing given a predictor

$$P(j|i) = a_{ij}/a_i \tag{A1}$$

where

$$a_i \equiv \sum_{j=1}^{n} a_{ij}. \tag{A2}$$

Note for a given predictor class $i$ that if $P(j|i) = 1/n$ ($n = 3$ for the icing case) for all $j$, then very little information is available to predict icing from that prediction class $i$. However, holding $i$ fixed, if $P(j_0|i)$ is near 1 for some $j_0$ and $P(j|i)$ is near 0 for all other $j$ values, then there is near-perfect predictability of icing class $j_0$ by the predictor class $i$. The potential predictability for a specific number of class intervals can be defined

$$\mathrm{PP} = \frac{n}{n-1}\sum_{i=1}^{m} P_1(i)\left\{\sum_{j=1}^{n}\left[P(j|i)-\frac{1}{n}\right]^2\right\} \tag{A3}$$

where

$$P_1(i) \equiv a_i/\sum_{j=1}^{n}\sum_{i=1}^{m} a_{ij} \tag{A4}$$

is the marginal probability of the $i$th predictor. Observe that PP is near 1 for near-perfect predictability as defined above, while PP is near 0 for little predictability [$P(j|i) \simeq 1/n$].

If PP were computed for 1000 repetitions with the value of the icing classes provided at random, for a given number of bivariate pairs and class intervals one would expect by chance that the 100 largest computed PP would be substantially greater than zero. In particular, as the number of predictor intervals approaches the number of observations we expect some PP to approach 1.0, since $P(j|i)$ will be 1.0 for some value of

$j$ at each given $i$ class. We therefore define the potential predictability PP of a data set as *significant at the 10% level* if

$$\mathrm{APP} \geqslant 0$$

where

$$\mathrm{APP} = \mathrm{PP} - \mathrm{PP}_{901} \qquad \text{(A5)}$$

is the absolute potential predictability. Here $\mathrm{PP}_{901}$ is the 901st value of (A3), in an ascending ordered sequence of PP values, compared by a Monte Carlo experiment with 1000 random trials with the same sample size and number of class intervals as the observational set.

In analogous manner, the computed $a_0$ value for three predictand categories could, in general, be greater than 0.3333 based upon a random data set. We define a score $a_0$ to be *significant at the 10% level* if

$$a_s \geqslant 0$$

where

$$a_s = a_0 - a_{901}. \qquad \text{(A6)}$$

Here $a_{901}$ is the 901st value of $a_0$, in an ascending ordered sequence of $a_0$ values computed by a Monte Carlo experiment, of 1000 random trials with the same sample and number of class intervals as the observational set.

## REFERENCES

Ackley, S. F., and M. K. Templeton, 1979: Computer modelling of atmospheric ice accretion. CRREL Rep. 79-4, Cold Regions Research and Engineering Laboratory, Hanover, 36 pp.

Brown, R. D., and T. A. Agnew, 1985a: Characteristics of marine icing in Canadian waters. *Proc. Int. Workshop on Offshore Winds and Icing,* Halifax, N.S., Environmental Canada, 78–94.

——, and ——, 1985b: Evaluation of currently available marine icing models for prediction of icing of ships and offshore structures. *Proc. Int. Workshop on Offshore Winds and Icing,* Halifax, N.S., Environment Canada, 123–139.

Borisenkov, E. P., and V. V. Panov, 1972: Fundamental results and perspectives in research into the hydrometeorological aspects of ship icing. Arkt. Antarkt. Nauchno-Issled Inst. T298, Leningrad, 5–33.

De Angelis, R. M., 1974: Superstructure icing. *Mar. Wea. Log,* **18,** 1–7.

Feit, D. M., 1985: Ship superstructure ice accretion guidance forecasts. *Proc. Int. Workshop on Offshore Winds and Icing,* Halifax, N.S., Environment Canada, 278–297.

George, D. J., 1975: The frequency of weather conditions favourable for ship spray icing on the seas round Iceland during the 1972–73 winter. *Mar. Obs.,* **45,** 177–185.

Hay, R. F. M., 1956: Ice accumulation on trawlers in northern waters. *Met. Mag.,* **85,** 225–229.

Jarvis, E. C., 1983: New version of the AES freezing spray forecast model. Internal Rep., Canadian Atmospheric Environment Service, Downsview. (Unpublished manuscript.)

Jessup, R. G., 1985: Forecasting techniques for ice accretion on different types of marine structures, including ships, platforms and coastal facilities. Marine Meteorology and Related Oceanographic Activities, Rep. 15, WMO, Canada, 90 pp.

Hoerner, I. S. F., 1965: *Fluid-Dyanmic Drag.* Hoerner Fluid Dynamics, Brick Town, N.J., 438 pp.

Kachurin, L. G., L. I. Gashin and I. A. Smirnov, 1974: Icing rate of small displacement fishing boats under various hydrometeorological conditions. *Meteorologiya i Gidrologiya,* **3,** Moscow, 50–60.

Kraus, E. B., 1972: *Atmosphere–Ocean Interaction,* Oxford University Press, London, 275 pp.

Lee, A., 1958: Ice accumulation on trawlers in the Barents sea. *Mar. Obs.,* **28,** 138–142.

List, R., 1977: Ice accretions on structures. *J. Glaciol.,* **19,** 451–465.

Lozowski, E. P., and E. M. Gates, 1985: An overview of marine icing modeling. *Proc. Int. Workshop on Offshore Winds and Icing,* xxx, Environment Canada, 102–122.

Lozowski, E. P., J. R. Stallabrass and P. F. Hearty, 1983: The icing of an unheated, non-rotating cylinder. Part I: A simulation model. *J. Climate Appl. Meteor.,* **22,** 2053–2062.

Lundqvist, J., and I. Udin, 1977: Ice accretion on ships with special emphasis on Baltic conditions. Winter Navigation Research Board, Res. Rep. 23. Swedish Meteorological and Hydrological Institute, 32 pp.

MacDonald, K. A., and R. G. Jessup, 1985: Evaluation of a freezing spray forecast system. *Proc. Int. Workshop on Offshore Winds and Icing,* xxx, Environment Canada, 267–277.

Makkonen, L., 1981: Estimating intensity of atmospheric ice accretion on stationary structures. *J. Applied Meteor.,* **20,** 595–600.

Makkonen, L., 1984: Modeling of ice accretion on wires. *J. Climate Appl. Meteor.,* **23,** 929–939.

Makkonen, L., and J. R. Stallabrass, 1985: The effect of roughness on the rate of ice accretion on a cylinder. *Ann. Glaciology,* **6,** 142–145.

Mertins, H. O., 1968: Icing on fishing vessels due to spray. *Mar. Obs.,* **38,** 128–130.

Minsk, L. D., 1977: Ice accumulation on ocean structures. CRREL Rep. 77-17, Cold Regions Research and Engineering Laboratory, Hanover, N.H., 42 pp.

Owen, P. R., and W. R. Thomson, 1963: Heat transfer across rough surfaces. *J. Fluid Mech.,* **15,** 321–334.

Panov, V. V., 1978: Icing of Ships. *Polar Geography,* **2,** 166–186.

Pease, C. H., and A. L. Comiskey, 1985: Vessel icing in Alaskan waters. NOAA Data Rep., ERL PMEL-14, Pacific Marine Environmental Laboratory, Seattle, 16 pp.

Preisendorfer, R. W., 1983: Proposed studies of some basic marine atmospheric visibility prediction schemes using model output statistics. [Unpublished manuscript. Available from NOAA/PMEL Library, 7600 Sand Point Way N.E., Seattle, WA 98115.]

Shellard, H. C., 1974: The meteorological aspects of ice accretion of ships. Reports on Marine Science Affairs, Rep. No. 10, WMO-No. 397, WMO, Geneva, 34 pp.

Stallabrass, J. R., 1980: Trawler icing: A complication of work done at N.R.C. Mech. Eng. Rep. MD-56, N.R.C. No. 19372, National Research Council, Ottawa, 103 pp.

Sawada, T., 1966: A forecasting method for ship icing near the Kuril Islands. *J. Meteor. Res.,* **18,** 665–673.

Sawada, T., 1973: Studies and predictions of ice accumulation on ships. *Bull. Hakodate Mar. Obs.,* **17,** 1–7.

Threlkeld, J. L., 1970: *Thermal Environmental Engineering,* Prentice-Hall, 495 pp.

Vasilyeva, G. V., 1971: Hydrometeorological conditions of the icing of sea-going ships. *Tr. Gidromettsentra SSR,* **87,** 82–92.

Wise, J. L., and A. L. Comiskey, 1980: Superstructure icing in Alaskan waters. NOAA Special Rep., Pacific Marine Environmental Laboratory, Seattle, WA, 30 pp. [NTIS PB81-135188.]

Zakrzewski, W. P., 1986: *Icing of ships. Part I: Splashing a ship with spray.* NOAA Tech. Memo., ERL PMEL-66, Pacific Marine Environmental Laboratory, Seattle, WA, 74 pp.